### Ada Lovelace's Computer Program Apple Time

Elsa Gonsiorowski

June 6, 2025

# Apple Time

20 Min Short Talk 15 Min Discussion / Breakout 10 Min Prizes!

- Links are in orange
- Full screen is recommended
- Slides available at gonsie.com/talks

### Elsa Gonsiorowski



- HPC I/O Support Specialist in Livermore Computing since 2016
- LC Hotline technical consultant, focused on user engagement and communication
- Working remotely in RI
- Excited about emacs, org-mode, static websites, fish shell, cmake, documentation, crossfit, rowing, knitting

### LC: Livermore Computing





### HPC Up Close poster session Join us on June 17, 2pm at the Central Cafe

### World's First Computer Program



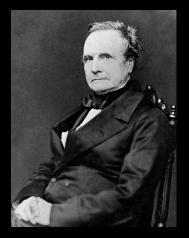
- History of the first computer
- Brief bio for Ada Lovelace
- Overview of some math
- Hypothesis: Ada Lovelace was the first computer programmer

### Mathematical Tables - 200 years ago



- Calculated values of logarithmic and trigonometric functions
- Built by hand by human "computers"
- Used to do rapid multiplication, division, and exponentiation

### Charles Babbage and the Difference Engine



### 1791–1871

- Idea for a Difference Engine to mechanically do the work of human computers
  - Began development in 1822
  - would have composed 25,000 parts, weighed 15 tons, stood 8 feet tall

# Analytical Engine

- Design began in 1833, described in 1837
- General purpose, i.e., Turing Complete
- Arithmetic logic unit, control flow (conditional branching and loops), memory, printer, and bell



# Augusta Ada King (née Byron), Countess of Lovelace

- Dec. 10, 1815–Nov. 27, 1852
- Child of poet Lord Byron and Lady Byron
- 1833: Met Charles
   Babbage at a party
- 1835: Married William King who became Earl of Lovelace



### Babbage and Young Lady Byron

- 1833: Met at a party (Babbage age 41, Ada age 17)
- Ada had extensive mathematics eduction to "ward off wild, romatic sensibility" of her father
- They were in the same social circle and wrote each other frequently

16 Jan 1844 Dean Bullage . The coat is safe, & mile ha Aclinent by Carlote at four house either on Thatay evening on or Sunday maring . A your up on Membery to get me texpines for Sunday. I am going to Jour very carly on Tueste morning , for one on two nights . \_ I hope to ver

### Sketch of the Analytical Engine

#### SCIENTIFIC MEMOIRS,

SELECTED FROM

THE TRANSACTIONS OF

#### FOREIGN ACADEMIES OF SCIENCE

#### AND LEARNED SOCIETIES,

AND FROM

#### FOREIGN JOURNALS.

#### EDITED BY

#### RICHARD TAYLOR, F.S.A.,

FELLOW OF THE LINNEAN, GEOLOGICAL, ASTRONOMICAL, ANATIC, STATISTICAL AND GEOGRAFHICAL SOCIETIES OF LONDON; HONORARY MEMBER OF THE NATURAL HISTORY SOCIETY OF MOSCOW.

#### VOL. III.

#### LONDON:

#### PRINTED BY RICHARD AND JOHN E. TAYLOR, RED LION COURT. PLANT STREET.

SILD INF LONGMAN, GENE, BLOWF, GREEF, AND LONGMANSI (ADELL) EIDGWAT AND BONSI BIKERWOON, GUILBERT, AND FIFE, INFIEL AND MARHAILL IN FFELLOWERI S. HIGHLEY; WHITTAKER AND GOI AND J. B. BAILLIERE, LONDON: —AND BY A. AND C. BLACK, AND THOMAS CLARK, BENDREGEN INFIT AND SON, GLANGOW — MILLIERD AND BONS, AND MODGES AND M'ARTHER, DEBLIN: —DOBBON, PHILADERIALI, AND GOODIGL AND WY YORK.

#### 1843

#### 666

#### ARTICLE XXIX.

Sketch of the Analytical Engine invented by Charles Babbage Esq. By L. F. MENABREA, of Turin, Officer of the Military Engineers.

[From the Bibliothèque Universelle de Génève, No. 82. October 1842.]

[BEFORE submitting to our readers the translation of M. Menabrea's memoir 'On the Mathematical Principles of the ANALY-TICAL ENORS' invented by Mr. Babbage, we shall present to them a list of the printed papers connected with the subject, and also of those relating to the Difference Engine by which it was preceded.

For information on Mr. Babbage's "Difference Engine," which is but slightly alluded to by M. Menabrea, we refer the reader to the following sources:---

 Letter to Sir Humphry Davy, Bart, P.R.S., on the Application of Machinery to Calculate and Print Mathematical Tables. By Charles Babbage, Eaq., F.R.S. London, July 1822. Reprinted, with a Report of the Council of the Royal Society, by order of the House of Commons, May 1823.

 On the Application of Machinery to the Calculation of Astronomical and Mathematical Tables. By Charles Babbage, Esq.—Memoirs of the Astronomical Society, vol. i. part 2. London, 1822.

3. Address to the Astronomical Society by Henry Thomas Colebrooke, Esq., F.R.S., President, on presenting the first Gold Medal of the Society to Charles Babbage, Eaq., for the invention of the Calculating Engine.—Memoirs of the Astronomical Society. London, 1822.

 On the Determination of the General Term of a New Class of Infinite Series. By Charles Babbage, Esq.—Transactions of the Cambridge Philosophical Society.

 On Mr. Babbage's New Machine for Calculating and Printing Mathematical Tables.—Letter from Francis Baily, Esq., F.R.S., to M. Schuimacher. No. 46, Astronomische Nachrichten. Reprinted in the Philosophical Magazine, May 1824.

6. On a Method of expressing by Signs the Action of Ma-

## Bernoulli Numbers

$$\sum n = \frac{1}{2}n^2 + \frac{1}{2}n$$

$$\sum n^2 = \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n$$

$$\sum n^3 = \frac{1}{4}n^4 + \frac{1}{2}n^3 + \frac{1}{4}n^2$$

$$\sum n^{m} = \frac{1}{m+1} (B_0 n^{m+1} \pm {\binom{m+1}{1}} B_1 n^{m} + {\binom{m+1}{2}} B_2 n^{m-1} + \dots)$$

$$A_{0} = -\frac{1}{2} \cdot \frac{2n-1}{2n+1}$$

$$A_{1} = \frac{2n}{2}$$

$$A_{3} = \frac{2n(2n-1)(2n-2)}{2 \cdot 3 \cdot 4}$$

$$A_{5} = \frac{2n(2n-1)(2n-2)(2n-3)(2n-4)}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6}$$

- Note A Promise of a machine that can perform arbitrary mathematical operations
- Note G Lady Lovelace's Objection despite it's power, the machine does not "think"
- Note D "Diagram of development" for calculating  $B_7 = -1(A_0 + B_1 A_1 + B_3 A_3 + B_5 A_5)$

[ .							Data						Working Variables.						Result Variables.			
Number of Operation	Nature of Operation.	Variables acted upon.	Variables receiving results.	Indication of change in the value on any Variable.	Statement of Results.	<sup>IV</sup> 1 000 1	<sup>1</sup> V <sub>2</sub> O 0 0 2 2	<sup>1</sup> V <sub>3</sub> О 0 4 П	°V4 0000	°V\$ 0000	°V° () 0 0 0 0	°V7 00000	9¥8 00000	°V <sub>9</sub> ○ 0 0 0	°V30 0000	<sup>0</sup> V <sub>11</sub> O 0 0 0	<sup>6</sup> V <sub>12</sub> O 0 0 0	°Y <sub>33</sub> O 0 0 0	$\begin{tabular}{c} B_1 & a & a \\ \end{tabular} B_1 & a & \end{tabular} \\ \end{tabular} fraction. \end{tabular} \end{tabular} \end{tabular} \end{tabular} \end{tabular} \end{tabular}$	E Bain a decimalOn fraction.	<sup>™</sup> B <sub>6</sub> in a decimal O <sup>™</sup> fraction.	<sup>o</sup> V <sub>21</sub> O 0 0 B <sub>7</sub>
1 2 3 4 5 6 7	1.+++1	${}^{1}V_{4} - {}^{1}V_{1}$ ${}^{1}V_{5} + {}^{1}V_{1}$ ${}^{2}V_{5} + {}^{2}V_{4}$ ${}^{1}V_{11} + {}^{1}V_{2}$ ${}^{0}V_{13} - {}^{2}V_{11}$	1V <sub>4</sub> , 1V <sub>5</sub> , 1V <sub>6</sub> 2V <sub>4</sub> 2V <sub>5</sub> 1V <sub>11</sub> 2V <sub>11</sub> 1V <sub>12</sub> 1V <sub>12</sub> 	$\begin{cases} {}^{1}V_{2} = {}^{1}V_{2} \\ {}^{1}V_{3} = {}^{1}V_{3} \\ {}^{1}V_{1} = {}^{2}V_{4} \\ {}^{1}V_{1} = {}^{1}V_{1} \\ {}^{1}V_{3} = {}^{2}V_{5} \\ {}^{1}V_{1} = {}^{1}V_{1} \\ {}^{1}V_{3} = {}^{1}V_{1} \\ {}^{2}V_{4} = {}^{0}V_{4} \\ {}^{1}V_{1} = {}^{2}V_{12} \\ {}^{1}V_{12} = {}^{1}V_{12} \\ {}^{2}V_{11} = {}^{0}V_{11} \\ {}^{0}V_{13} = {}^{1}V_{13} \\ {}^{1}V_{3} = {}^{1}V_{3} \\ {}^{1}V_{3} \\ {}^{1}V_{3} = {}^{1}V_{3} \\ {}^{1$	$\begin{array}{l} = 2 \; n \\ = 2 \; n - 1 \\ = 2 \; n + 1 \\ = 2 \; n + 1 \\ = \frac{2 \; n - 1}{2 \; n + 1} \\ = \frac{1 \; 2 \; n - 1}{2 \; \cdot \; 2 \; n - 1} \\ = \frac{1 \; 2 \; n - 1}{2 \; \cdot \; 2 \; n + 1} = \Lambda_0 \\ = \; n - 1 \; (= 3) \end{array}$		2	n    n	2 n 2 n = 1  0  	2 n 2 n+ 1 0  	2 n				  n - 1	$\frac{\frac{2n-1}{2n+1}}{\frac{1}{2}\cdot\frac{2n-1}{2n+1}}$		$-\frac{1}{2}\cdot\frac{2n-1}{2n+1}=\lambda_0$				
8 9 10 11 12	+ × + -	${}^{1}V_{6} + {}^{1}V_{7}$ ${}^{1}V_{21} \times {}^{3}V_{11}$ ${}^{1}V_{12} + {}^{1}V_{13}$ ${}^{1}V_{12} + {}^{1}V_{13}$	<sup>1</sup> V <sub>12</sub> <sup>2</sup> V <sub>13</sub> <sup>2</sup> V <sub>10</sub>	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{l} = 2 + 0 = 2 \\ = \frac{2n}{2} = \lambda_1 \\ = B_1 \cdot \frac{2n}{2} = B_1 \lambda_1 \\ = -\frac{1}{2} \cdot \frac{2n-1}{2n+1} + B_1 \cdot \frac{2n}{2} \\ = u - 2(=2) \end{array}$		2				 2 n 	2 2			  n - 2	$\frac{\frac{2}{2}n}{\frac{2}{2}} = \Lambda_1$ $\frac{\frac{2}{2}n}{\frac{2}{2}} = \Lambda_1$ $\dots$	$B_1, \frac{2n}{2} = B_1 A_1$	$\left\{-\frac{1}{2},\frac{2n-1}{2n+1}+B_1,\frac{2n}{2}\right\}$	B1			
13 14 15 16 17 18, 19 20 21 22 23	+++×++×	${}^{1}V_{1} + {}^{1}V_{7}$ ${}^{2}V_{6} + {}^{2}V_{7}$ ${}^{1}V_{8} \times {}^{3}V_{11}$ ${}^{2}V_{6} - {}^{1}V_{1}$ ${}^{1}V_{1} + {}^{2}V_{7}$ ${}^{3}V_{6} + {}^{3}V_{7}$ ${}^{1}V_{9} \times {}^{4}V_{11}$	<sup>2</sup> V <sub>7</sub> <sup>1</sup> V <sub>8</sub> <sup>4</sup> V <sub>11</sub> <sup>3</sup> V <sub>6</sub> <sup>3</sup> V <sub>7</sub> <sup>4</sup> V <sub>9</sub> <sup>4</sup> V <sub>11</sub>	$ \begin{cases} {}^{1}V_{9} = {}^{0}V_{9} \\ {}^{4}V_{11} = {}^{5}V_{11} \\ {}^{1}V_{22} = {}^{1}V_{22} \end{cases} $	$\begin{array}{l} =2n-1\\ =2+1=3\\ \ldots\\ =\frac{2}{3}, =-1\\ =\frac{2}{3}, \frac{2}{3}, \ldots\\ =\frac{2}{3}, \frac{2}{3}, \ldots\\ =\frac{2}{3}, -2\\ =\frac{2}{3}, -2\\ =\frac{2}{3}, -2\\ =\frac{2}{3}, -2\\ =\frac{2}{3}, \frac{2}{3}, \ldots\\ =\frac{2}{3}, \frac{2}{3}, \ldots\\ =\frac{2}{3}, \frac{2}{3}, \ldots\\ =\frac{2}{3}, \frac{2}{3}, \ldots\\ =n, \frac{2}{3}, \frac{2}{3}, \ldots\\ =n, \frac{2}{3}, \frac{2}{3}, \ldots\\ =n, \frac{2}$	 . 1 . 1 					2 n - 1  2 n - 1  2 n - 2  2 n - 2    	a 4 4 …			  n - 3	1	B <sub>2</sub> A <sub>3</sub> 0	$\left\{ \Lambda_{2}+B_{1}\Lambda_{1}+B_{2}\Lambda_{3}^{'}\right\}$		B <sub>3</sub>		
24 25	+	"V <sub>13</sub> +°V <sub>2</sub>	4 <sup>1</sup> V <sub>24</sub>	$ \begin{cases} {}^{4}V_{13} = {}^{6}V_{13} \\ {}^{6}V_{23} = {}^{1}V_{24} \\ {}^{1}V_{1} = {}^{1}V_{1} \\ {}^{1}V_{3} = {}^{1}V_{3} \\ {}^{4}V_{6} = {}^{6}V_{6} \\ {}^{6}V_{7} = {}^{6}V_{7} \end{cases} $	$= B_7$ $= n + 1 = 4 + 1 = 5$ by a Variable-card. by a Variable card.					ows a re	petition 0	of Oper 0		hirteen	to twent	y-three.						В,

#### Diagram for the computation by the Engine of the Numbers of Bernoulli. See Note G. (page 722 et seq.)

1	é						Data.						Working Variables.							Result Variables.			
Number of Operatio	Nature of Operation.	Variables acted upon. Variables receiving results.	Indication of change in the value on any Variable.	Statement of Results.	$V_1 \\ \bigcirc \\ 0 \\ 0 \\ 0 \\ 1 \\ 1$	<sup>1</sup> V <sub>2</sub> O 0 0 2 2	<sup>1</sup> V <sub>3</sub> 0 0 4 π	°V4 0000	°Vs 0000	°V° ○ 0 0 0 0	°V7 0000	940000	<sup>0</sup> Y <sub>9</sub> O 0 0 0	°Y 30 0 0 0 0	<sup>0</sup> V <sub>11</sub> O 0 0 0	<sup>6</sup> V <sub>12</sub> O 0 0 0	°Y <sub>11</sub> O 0 0 0	Ta Bi in a decimal Or fraction.	E Bain a GecimalOR	<sup>6</sup> E <sup>b</sup> in a decimalO <sup>13</sup> fraction.	<sup>o</sup> V <sub>21</sub> O 0 0 B <sub>7</sub>		
1 2 3	× +	$1V_2 \times 1V_3$ $1V_4 - 1V_1$ $1V_5 + 1V_1$	1V <sub>4</sub> , 1V <sub>5</sub> , 1V <sub>6</sub> 2V <sub>4</sub>	$ \left\{ \begin{array}{l} {}^{1}\mathrm{V}_{2} = {}^{1}\mathrm{V}_{2} \\ {}^{1}\mathrm{V}_{3} = {}^{1}\mathrm{V}_{3} \\ {}^{1}\mathrm{V}_{4} = {}^{2}\mathrm{V}_{4} \\ {}^{1}\mathrm{V}_{1} = {}^{1}\mathrm{V}_{1} \\ {}^{1}\mathrm{V}_{5} = {}^{2}\mathrm{V}_{5} \\ {}^{1}\mathrm{V}_{1} = {}^{1}\mathrm{V}_{1} \end{array} \right\} $	$= 2\pi$ = $2\pi - 1$ = $2\pi + 1$	1 1	2	n 	2 n 2 n - 1	2 n 2 n+1	2 n						ill's						
4 5 6 7	+ -	${}^{2}V_{5} + {}^{2}V_{4}$ ${}^{1}V_{11} + {}^{1}V_{2}$ ${}^{0}V_{13} - {}^{2}V_{11}$		$\begin{cases} {}^{2}V_5 = {}^{0}V_5 \\ {}^{2}V_4 = {}^{0}V_4 \\ \\ {}^{1}V_{11} = {}^{2}V_{11} \\ {}^{1}V_2 = {}^{1}V_2 \\ \\ {}^{2}V_{11} = {}^{0}V_{11} \\ {}^{0}V_{13} = {}^{1}V_{13} \end{cases}$	$ = \frac{2n-1}{2n+1} \\ = \frac{1}{2} \cdot \frac{2n-1}{2n+1} \\ = -\frac{1}{2} \cdot \frac{2n-1}{2n+1} = \Lambda_0 \\ = n-1 (=3) $		2	  R	0  	0  						$ \frac{\frac{2n-1}{2n+1}}{\frac{1}{2} \cdot \frac{2n-1}{2n+1}} $ 0		$-\frac{1}{2} \cdot \frac{2n-1}{2n+1} = h_0$		10.19			
8 9 10 11	+ + ×	$V_2 + V_7$ $V_6 + V_7$ $V_8 + V_7$ $V_{11} \times V_{11}$	<sup>1</sup> V <sub>10</sub> <sup>1</sup> V <sub>7</sub> <sup>3</sup> V <sub>11</sub> <sup>1</sup> V <sub>12</sub>	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	$ \begin{array}{l} = 2 + 0 = 2 & \dots \\ = \frac{2 + 0}{2} = \lambda_1 & \dots \\ = B_1 \cdot \frac{2 + n}{2} = B_1 \Lambda_1 & \dots \\ = -\frac{1}{2} \cdot \frac{2 + n - 1}{2 + n + 1} + B_1 \cdot \frac{2 + n}{2} & \dots \end{array} $		2				 2 n 	 2 2 			n - 1 		$B_1, \frac{2\pi}{2} = B_1 A_1$		B <sub>1</sub>				
12 13 14	-	<sup>1</sup> V <sub>10</sub> - <sup>1</sup> V <sub>1</sub>	<sup>2</sup> V <sub>10</sub> <sup>2</sup> V <sub>6</sub> <sup>2</sup> V <sub>7</sub>	$ \begin{cases} {}^{1}V_{16} = {}^{2}V_{29} \\ {}^{1}V_{1} = {}^{1}V_{1} \\ \\ \\ {}^{1}V_{6} = {}^{2}V_{6} \\ {}^{1}V_{1} = {}^{1}V_{1} \\ {}^{1}V_{1} = {}^{1}V_{1} \\ \\ \\ {}^{1}V_{1} = {}^{1}V_{1} \\ \\ \\ \\ \\ V_{2} = {}^{2}V_{2} \end{cases} $	= n - 2 (= 2) = 2n - 1 = 2 + 1 = 3	1 1 1 1					 2 n - 1 				 n – 2		0	$\left\{-\frac{1}{2},\frac{2n-1}{2n+1}+B_1,\frac{2n}{2}\right\}$	11 mm				
15 16 17 18,	l×		<sup>1</sup> V <sub>8</sub> <sup>4</sup> V <sub>11</sub> <sup>3</sup> V <sub>6</sub> <sup>3</sup> V <sub>7</sub>	$\begin{cases} 2V_{6} = 2V_{6} \\ 2V_{7} = 2V_{7} \\ 3V_{11} = 4V_{11} \\ 2V_{6} = 3V_{6} \\ 4V_{1} = 1V_{1} \\ 2V_{7} = 3V_{7} \\ 4V_{7} = 4V_{7} \end{cases}$	$= \frac{2n-1}{3} \\ = \frac{2n}{2} \cdot \frac{2n-1}{3} \\ = 2n-2 \\ = 3+1=4 $						2 n - 1  2 n - 2 		$\frac{2n-1}{3}$ 0			$\frac{2n}{2}, \frac{2n-1}{3}$				alter a			
19 20 21 22	Lx ×	<sup>1</sup> V <sub>9</sub> × <sup>4</sup> V <sub>1</sub> ; <sup>1</sup> V <sub>22</sub> × <sup>5</sup> V <sub>1</sub> ;	ov12	$\begin{cases} {}^{3}V_{7} = {}^{3}V_{7} \\ {}^{1}V_{9} = {}^{6}V_{9} \\ {}^{4}V_{11} = {}^{5}V_{11} \\ {}^{1}V_{22} = {}^{1}V_{22} \\ {}^{6}V_{12} = {}^{2}V_{12} \\ {}^{2}V = {}^{6}V \end{cases}$	$ \begin{array}{l} \cdot = \frac{2  n - 2}{4} & \dots \\ \cdot = \frac{2  n}{2} \cdot \frac{2  n - 1}{3} \cdot \frac{2  n - 2}{4} = \Lambda_3 \\ \cdot = \mathrm{B}_3 \cdot \frac{2  n}{2} \cdot \frac{2  n - 1}{3} \cdot \frac{2  n - 2}{3} = \mathrm{B}_3  \mathrm{A} \end{array} $						2 n - 2 			$\frac{2n-5}{4}$ 0		$ \begin{cases} \frac{2n}{2}, \frac{2n-1}{3}, \frac{2n-2}{3} \\ = A_3 \end{cases} $	B <sub>2</sub> A <sub>3</sub>			Ba		1	
23	- 1	<sup>2</sup> V <sub>10</sub> - <sup>1</sup> V <sub>1</sub>	<sup>3</sup> V <sub>10</sub>	$\left  \left\{ {}^{2V_{10} = 3V_{10}}_{1V_1 = 1V_1} \right\} \right $	$\begin{vmatrix} A_0 + B_1 \Lambda_1 + B_3 \Lambda_3 & \dots \\ = n - 3 (= 1) & \dots \\ \end{vmatrix}$	1		   	Iere foll	 	petition	of Oper	rations t	 	n - 3 to twent	ty-three.	0	$\left  \left\{ A_3 + B_1 A_1 + B_2 A_3 \right\} \right $	The second			and a	
24	+	"V13+"V2	W 24	$\left  \begin{cases} {}^{4}V_{13} \! = \! {}^{0}V_{13} \\ {}^{0}V_{24} \! = \! {}^{1}V_{24} \\ {}^{1}V_{1} \! = \! {}^{1}V_{1} \\ {}^{1}V_{3} \! = \! {}^{1}V_{3} \end{cases} \right $	$= B_7 \dots$			 n + 1			0	0										B,	

#### Diagram for the computation by the Engine of the Numbers of Bernoulli. See Note G. (page 722 et seq.)

 $\begin{cases} 4V_6 = {}^{6}V_6 \\ 4V_7 = {}^{0}V_7 \end{cases}$  by a Variable-card. by a Variable card. Program Snippet (GitHub Gist)

// ----- A0 -----/\* 01 \*/ v4 = v5 = v6 = v2 \* v3; // 2n
/\* 02 \*/ v4 = v4 - v1; // 2n - 1
/\* 03 \*/ v5 = v5 + v1; // 2n + 1

// In Lovelace's diagram, the below appears as v5 / v4, wl
/\* 04 \*/ v11 = v4 / v5; // (2n - 1) / (2n +

// On the first loop this calculates B3A3 and adds it on  $\gamma$  // On the second loop this calculates B5A5 and adds it on while (v10 > 0) {

// ----- B3A3, B5A5 ------

- What Did Ada Lovelace's Program Actually Do? TwoBitHistory.org
- Sketch of the Analytical Engine
- Translation of Note D to C. (gist)
- Wikipedia

- Introduce yourself to your group; what are you working on this summer?
- What is the most difficult bug you've encountered?
- What is the best thing you've attended so far this summer? And/or what are you most looking forward to?

### Prizes!

A prize will be awarded to anyone who shares their bug story at the end of the hour

# Created with Emacs, Org Mode, and $\ensuremath{\mathbb{E}} T_EX/\ensuremath{\mathbb{B}} T_ex$ . View the source.